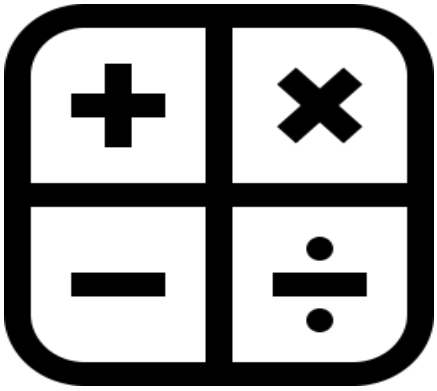
Written Calculation
Policy for Southwark Primary Schools



(Aligned with the 2014 National Curriculum)

Diane Andrews, Maths Consultant

Revised April 2017

Progression towards a standard written method of calculation

INTRODUCTION

This calculation policy has been written in line with the programmes of study taken from the revised **National Curriculum for Mathematics (2014)**.
It provides guidance on appropriate calculation methods and progression.
The content is set out in yearly blocks under the following headings: addition, subtraction, multiplication and division.
Statements taken directly from the programmes of study are listed in bold at the beginning of each section.

Knowing number facts and a secure knowledge of place value underpins the ability to calculate and these skills should be taught regularly.

Children should use mental methods as their first port of call, when appropriate, but for calculations that they cannot do mentally, they will need to use an efficient written method accurately and with confidence.

AIMS OF THE POLICY

- To ensure consistency and progression in our approach to calculation
- To ensure that children develop an efficient, reliable, formal written method of calculation for all operations
- To ensure that children can use these methods accurately with confidence and understanding

HOW TO USE THIS POLICY

- Use the policy as the basis of your planning but ensure you use previous or following years' guidance to allow for personalised learning
- Always use AfL to identify suitable next steps in calculation for groups of children
- If, at any time, children are making significant errors, return to the previous stage in calculation
- Always use suitable resources, models and images to support children's understanding of calculation and place value, as appropriate
- Ensure that children make estimates and use inverse operations to check calculations
- Encourage children to make sensible choices about the methods they use when solving problems

Diane Andrews, maths consultant (diane@countonmeconsultancy.co.uk)

Stages in Addition

Addition - Early Stages (EYFS)

Children will engage in a wide variety of songs and rhymes, games and activities.
They will begin to relate addition to **combining two groups of objects**, first by **counting all** and then by **counting on** from the largest number.
They will find one more than a given number.
In practical activities and through discussion they will begin to use the vocabulary involved in addition.

'You have five apples and I have three apples. How many apples altogether?'

Addition - Year One

- **Given a number, identify one more**
- **Read, write and interpret mathematical statements involving addition (+) and the equals (=) sign**
- **Add one- digit and two-digit numbers within 20, including zero**
- **Solve missing number problems e.g. 10 + □ = 16**

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Children should continue to practise counting on from any number e.g. 'Put five in your head and count on four.'

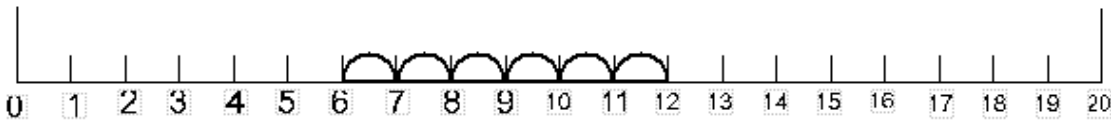
Initially use a **number track** to count on for addition, counting on from the largest number:

5 + 4 = 9

'Put your finger on number five. Count on (count forwards) four.'

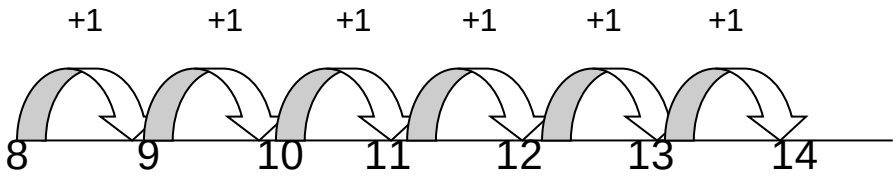
Then progress to a **marked number line:**

6 + 6 = 12



'Put your finger on number six and count on six.'

8 + 6 = 14 ‘Put your finger on number eight and count/ jump on six.’



Continue to practise counting on from the largest number for addition with totals within 20.

When children are ready, introduce calculations with totals beyond 20
e.g. 18 + 6 = 24

Ensure children are confident with using a marked number line before moving on to an empty number line (see Y2 guidance).

Addition - Year Two

- **Add numbers using concrete objects, pictorial representations, and mentally, including:**
    - **A two digit number and ones**
    - **A two digit number and tens**
    - **Two two-digit numbers**
    - **Three one-digit numbers (using a mental method)**

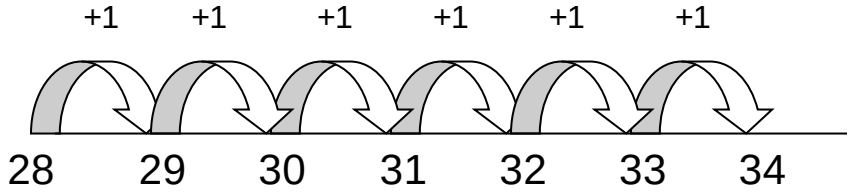
NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Consolidate counting on using a **marked number line.**

Then…

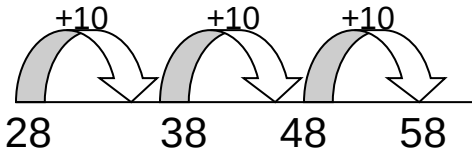
Count on in ones using an **empty number line,** within 100…

28 + 6 = 34



…and then in tens, within 100

28 + 30 = 58

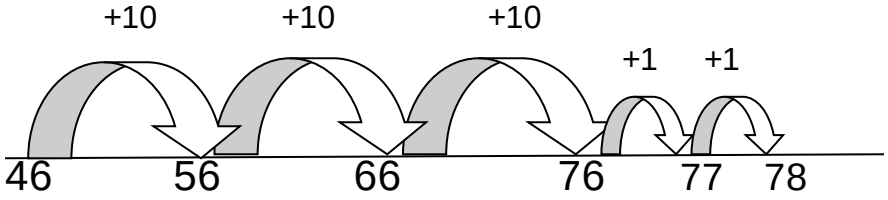


Use in conjunction with a **100 square** to show jumps of tens.

Add 2 two-digit numbers, within 100

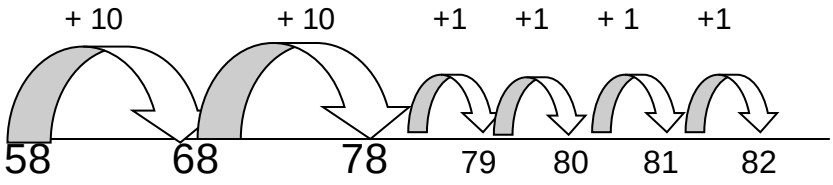
46 + 32 = 78

'Put the largest number first (46), and then partition the smaller number (32 = 30 + 2) and count on: 46 + 30 + 2.'



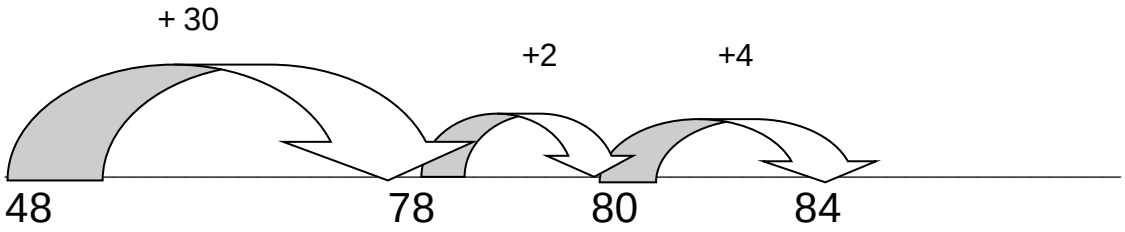
Use in conjunction with a **100 square** to show jumps of tens and ones.

58 + 24 = 82



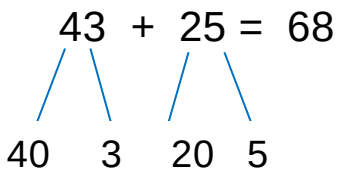
When children are confident, encourage more efficient jumps…

48 + 36 = 84



Use in conjunction with a **100 square** to show jumps of tens and ones/units.

The **partitioning method** is an alternative method for adding 2 two-digit numbers:



40 + 20 = 60
3 + 5 = 8
60 + 8 = 68

'Partition the numbers into tens and ones.
Add the tens together and then add the ones together.
Recombine to give the total'.

Use in conjunction with base ten apparatus (e.g. Dienes) to support understanding.

When children are confident, move on to calculations that bridge tens:

$$48 + 36 = 40 + 8 + 30 + 6$$

$$40 + 30 = 70$$
$$8 + 6 = 14$$
$$70 + 14 = 84$$

$$48 + 36 = 84$$

This is an alternative way of recording the partitioning method.
Continue to use base ten apparatus to support understanding.

When children are confident, further develop addition of 2 two-digit numbers with totals greater than 100, using a **200 grid** to support (see Y3 guidance).

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

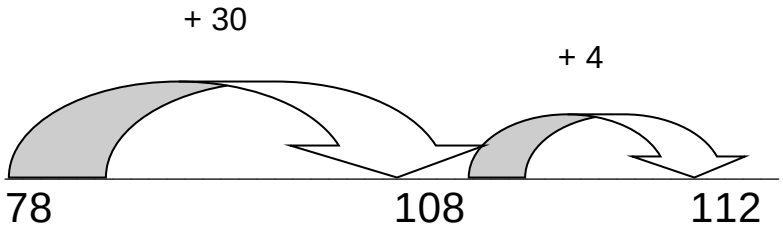
Addition - Year Three

- **Add numbers with up to three digits, using formal written method of columnar addition**

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Further develop the use of the **empty number line** with calculations that bridge 100:

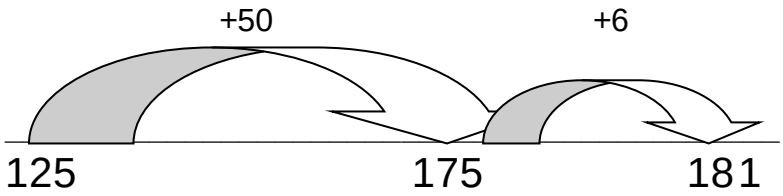
78 + 34 = 112



Use a **200 grid** to support counting on in tens when bridging 100

Further develop with addition of a three-digit and a two -digit number:

125 + 56 = 181



Further develop the **partitioning method** with calculations that bridge 100:

85 + 37 = 80 + 5 + 30 + 7

80 + 30 = 110
5 + 7 = 12
110 + 12 = 122

85 + 37 = 122

Consider the use of base ten apparatus (e.g. Dienes) to support understanding.

The partitioning method can also be used with three-digit numbers.

Introduce the **expanded written method** with the calculation presented both horizontally and vertically (in columns).

Initially use calculations where it is not been necessary to bridge across the tens or hundreds:

63 + 32 = 95

$$\begin{array}{rl} & 60 + 3 \\ + & \underline{30 + 2} \\ & 90 + 5 = 95 \end{array}$$

'Partition the numbers into tens and ones/units. Add the tens together and then add the ones/units together.
Recombine to give the total.'

Then refine the recording…

$$\begin{array}{rll} & 6\ 3 & \\ + & \underline{3\ 2} & \\ & 5 & (3 + 2) \\ + & \underline{9\ 0} & (60 + 30) \\ & 9\ 5 & \end{array}$$

Add the least significant digits (units) together first and then the tens in preparation for the formal written method.

Use base ten apparatus to support understanding.

This will lead into the **formal written method…**

$$\begin{array}{rl} & 6\ 3 \\ + & \underline{3\ 2} \\ & 9\ 5 \end{array}$$

Use the language of place value to ensure understanding:

'Three add two equals five. Write five in the ones/units column.
60 add 30 equals 90. Write 9 (90) in the tens column.
The total is 95.

NB Informal/mental methods, such as counting on an empty number line or partitioning, would be more appropriate for numbers of this size, but use two-digit numbers, initially, when introducing the columnar method.

Then introduce calculations where it is necessary to bridge, returning to an expanded method initially:

68 + 24 = 92

$$\begin{array}{rl} & 60 + 8 \\ + & \underline{20 + 4} \\ & 80 + 12 = 92 \end{array}$$

'Partition the numbers into tens and ones/units.
Add the tens together and then add the ones/units together.
Recombine to give the total (92).'

Then refine the recording…

```
  6 8
+ 2 4
  1 2   (8 + 4)
+ 8 0   (60 + 20)
  9 2
```

Add the least significant digits (units) together first and then the tens in preparation for the formal written method.

When children are ready, introduce the **formal written method,** where it is necessary to 'carry' ten from the units to the tens column:

```
  6 8
+ 2 4
  9 2
  1
```

Use the language of place value to ensure understanding:

'Eight add four equals 12. Write two in the units column and 'carry' one ten (10) across into the tens column. 60 add 20 and the ten that we 'carried' equals 90. Write 9 (90) in the tens column. 92 is the answer'.

The digit that has been 'carried' should be recorded under the line in the correct column.

When children are confident**,** extend with examples where it is necessary to bridge across the tens and the hundreds, returning to an expanded method, if necessary:

76 + 47 = 123

```
  70 + 6
+ 40 + 7
110 +13 = 123
```

'Partition the numbers into tens and ones/units. Add the tens together and then add the ones/units together. Recombine to give the answer.'

Then…

```
  7 6
+ 4 7
  1 3   (7 + 6)
+ 1 1 0 (70 + 40)
  1 2 3
```

Add the least significant digits (units) together first and then the tens in preparation for the formal written method.

When children are ready introduce the **formal written method,** where it is necessary to 'carry' across the columns, including calculations with a total greater than 100:

76 + 47 = 123

```
  4 7
+ 7 6
-----
1 2 3
1 1
```

Use the language of place value to ensure understanding:

'Seven add six equals 13. Write three in the units column and 'carry' one (10) across into the tens column.
40 add 70 and the ten that we 'carried' equals 120.
Write 2 (20) in the tens column and 'carry' one (100) across into the hundreds column (100).
The total is 123'.

The digits that have been 'carried' should be recorded under the line in the correct column.

When children are confident, further develop with the addition of 2 three- digit numbers:

178 + 243 = 421

```
  1 7 8
+ 2 4 3
-------
  4 2 1
  1 1
```

Continue to use base ten apparatus to support, if necessary.

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Addition - Year Four

- **Add numbers with up to 4 digits using the formal written method of columnar addition, where appropriate**

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to teach the use of **empty number lines** with three and four digit numbers, as appropriate.

Further develop the formal written method of addition, with three-digit numbers. Revisit the **expanded method** first, if necessary:

476 + 147 = 623

```
  4 7 6
+ 1 4 7
  ------
    1 3        (7 + 6)
+ 1 1 0        (70 + 40)
  5 0 0        (400 + 100)
  -----
  6 2 3
```

This will lead into the **formal written method…**

476 + 147 = 623

```
  4 4 7
+ 1 7 6
  -----
  6 2 3
  1 1
```

Use the language of place value to ensure understanding:

'Seven add six equals 13. Write three in the units column and 'carry' one across into the tens column (10).
40 add 70 and the ten that we carried equals 120. Write 2 in the tens column (20) and 'carry' 1 across into the hundreds column (100).
400 add 100 and the 100 that has been carried equals 600. Write 6 in the hundreds column (600). The total is 623.

The digits that have been 'carried' should be recorded under the line in the correct column.

When children are confident, introduce addition of a four-digit number and a three-digit number:

1,845 + 526 = 2,371

```
  1 8 4 5
+   5 2 6
  ------
  2 3 7 1
    1   1
```

Continue to develop with addition of 2 four-digit numbers and with decimal numbers (in the context of money or measurement).

£45·65 + £ 28·50 = £74·15

```
   45·65
+  28·50
   ------
   74·15
    1 1
```

Ensure that the decimal points line up.

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Addition - Year Five

- **Add whole numbers with more than 4 digits, including using a formal written method (columnar addition)**

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to teach the use of **empty number lines** with larger numbers and decimal numbers, as appropriate.

Continue to develop the **formal written method for addition** with larger numbers (and decimal numbers) and with the addition of three or more numbers:

21,848 + 1,523 = 23,371

```
  21848
+ 1523
 23371
  1 1
```

Continue to use the language of place value to ensure understanding.

Ensure that the digits that have been 'carried' are recorded under the line in the correct column.

Use **the formal written method** for the addition of decimal numbers:

£154·75 + £233·82 = £388·57

```
  154·75
+ 233·82
 388·57
   1
```

Ensure that the decimal points line up.

Continue to practise and apply the formal written method throughout Y5, including the addition of more than two numbers.

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Addition - Year Six

No objectives have been included in the programmes of study explicitly related to written methods for addition in Y6.

However, there is an expectation that children will continue to practise and use the **formal written method for larger numbers and decimals** and use these methods when solving problems, when appropriate (see previous year's guidance for methods).

Our aim is that by the end of Y6, children **use mental methods (with jottings)** when appropriate, but for calculations that they cannot do mentally, they use an efficient **formal written method** accurately and with confidence.

Stages in Subtraction

Subtraction - Early Stages (EYFS)

Children will engage in a variety of counting songs and rhymes and practical activities.
In practical activities and through discussion they will begin to use the vocabulary associated with subtraction.
They will find one less than a given number.
They will begin to relate subtraction to 'taking away' **using objects** to count 'how many are left' after some have been taken away.

'I have six apples. I take two apples away. How many are left?'
'Six take-away two leaves four'.

Children will begin to count back from a given number.

Subtraction - Year One

- **Given a number, identify one less**
- **Read, write and interpret mathematical statements involving subtraction (-) and the equals (=) sign**
- **Subtract one- digit and two-digit numbers within 20, including zero**
- **Solve missing number problems e.g. 20 - ☐ = 15**

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Children will continue to practise counting back from a given number.

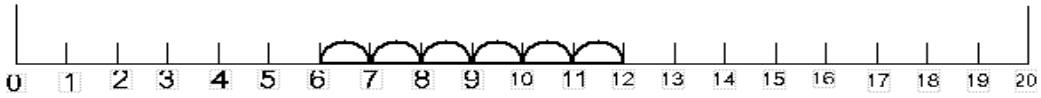
Initially use a **number track** to **count back** for subtraction:

9 – 5 = 4

'Put your finger on number nine. Count back five.'

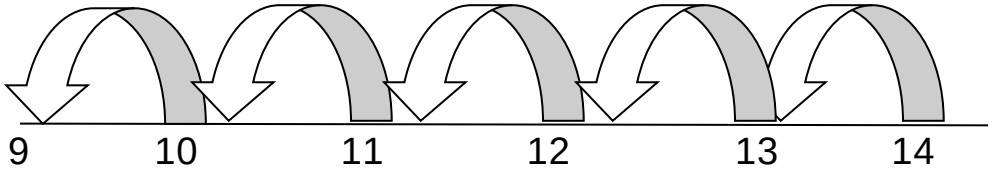
Then progress to a **marked number line:**

12 - 6 = 6



'Put your finger on number twelve and count back six.'

14 – 5 = 9



'Put your finger on number 14 and count/jump back five.'

Continue to practise counting back for subtraction with numbers within 20, and then when children are ready, with numbers beyond 20.

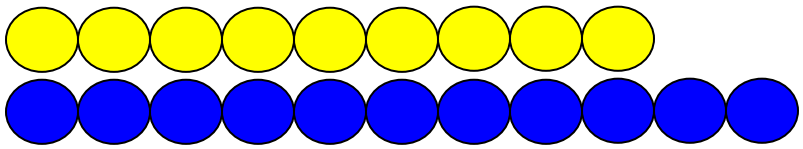
NB Ensure children are confident with using a **marked number line** before moving on to an empty number line (see Y2 guidance).

Counting on to find a small difference:

Introduce complementary addition to find differences (only use for **small** differences). The use of models is extremely important here to understand the idea of "difference".

Count up from the smallest number to the largest to **find the difference** using resources, e.g. cubes, beads, number tracks/lines:

11 – 9 = 2



The **difference between** nine and eleven is two.

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Subtraction - Year Two

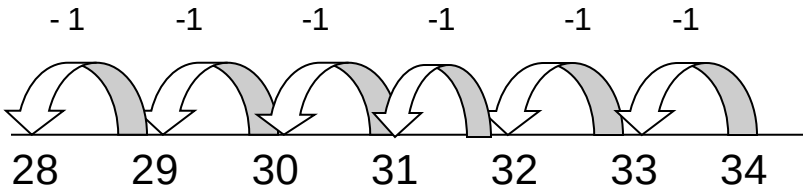
- **Subtract numbers using concrete objects, pictorial representations, and mentally, including:**
    - **A two digit number and ones**
    - **A two digit number and tens**
    - **Two two-digit numbers**

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.
Consolidate counting back using a **marked number line.**

Then…

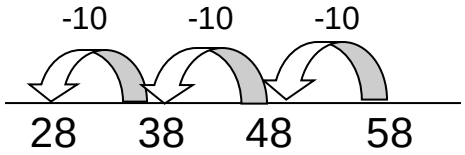
Count back using an **empty number line** within 100, in ones…

34 - 6 = 28



…and in tens:

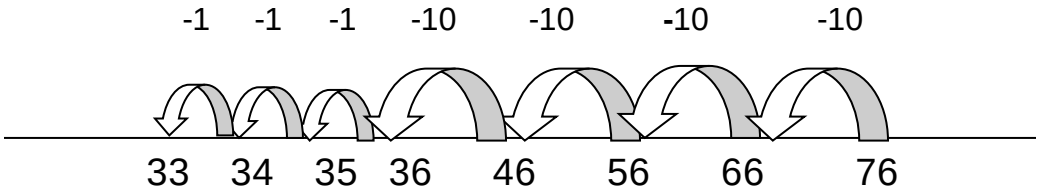
58 - 30 = 28



Use in conjunction with a **100 square** to show jumps of tens.

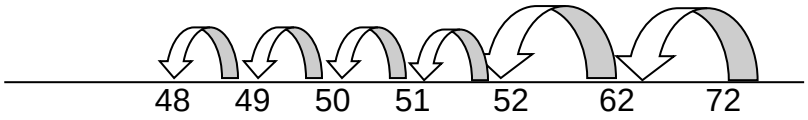
Count back on an **empty number line** to subtract 2 two-digit numbers, within 100:

76 – 43 = 33 (partition 43 into 40 + 3)



Use in conjunction with a **100 square** to show jumps of tens and ones.
Consider the use of base ten apparatus (e.g. Dienes) to support understanding.

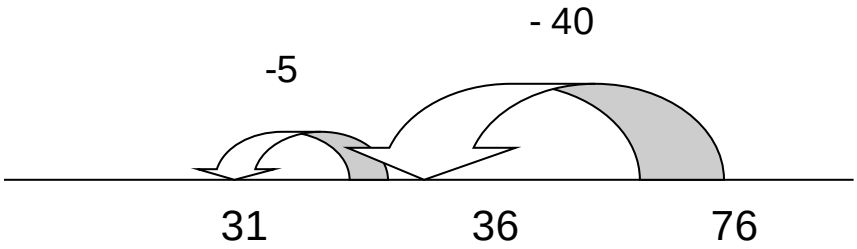
72 – 24 = 48



In this example regrouping is required (it has been necessary to bridge the tens).

If children are confident, use more efficient jumps:

76 – 45 = 31



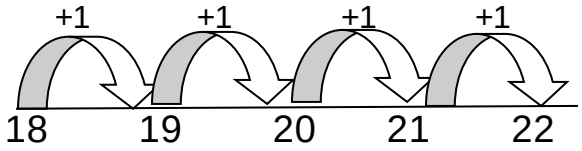
Use in conjunction with a **100 square** to show jumps of tens and ones.

Counting on to find a small difference

Use complementary addition to find differences (only use for **small** differences).
The use of models is extremely important here to understand the idea of "difference" (see Y1 guidance).

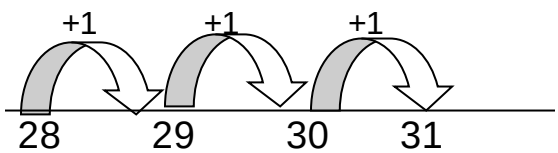
Count up from the smallest number to the largest to **find the difference**.

22 - 18 = 4



'The difference between 18 and 22 is 4.'

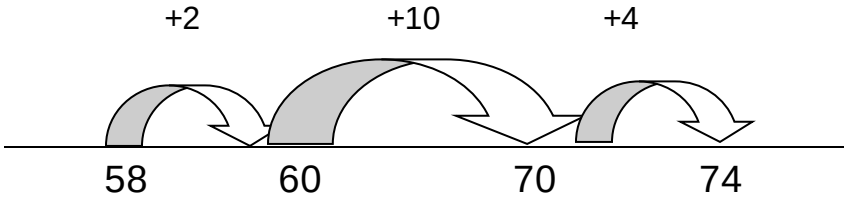
31- 28 = 3



'The difference between 28 and 31 is 3.'

If children are confident, further develop this method to find the difference, using more efficient jumps:

74 – 58 = 16



'The difference between 58 and 74 is 16.'

Further develop subtraction with numbers that bridge 100, using a **200 grid** to support (see Y3 guidance).

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

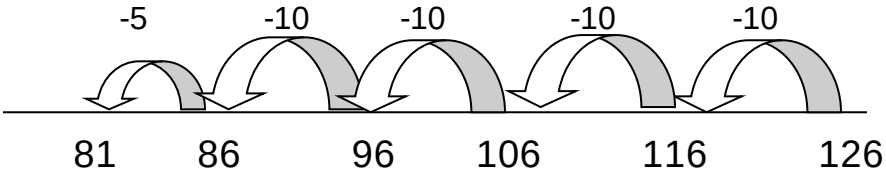
Subtraction - Year Three

- **Subtract numbers with up to three digits, using formal written method of columnar subtraction**

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

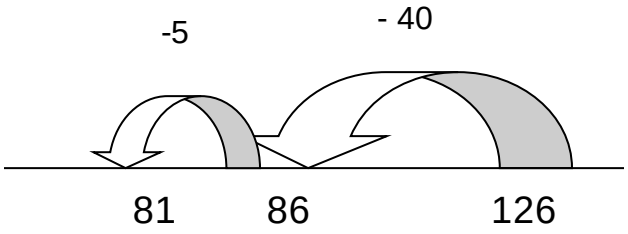
Further develop the use of the **empty number line** with calculations that **bridge 100:**

126 – 45 = 81



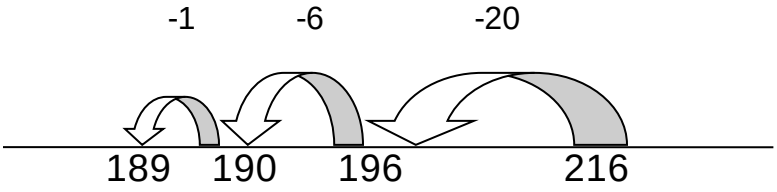
Use a **200 grid** to support counting back in tens and bridging 100

Then use more efficient jumps:



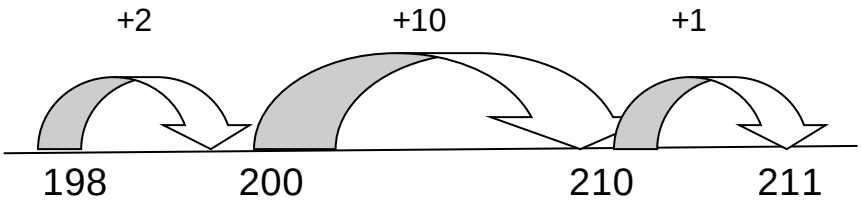
Extend with larger numbers by counting back…

216 – 27 = 189



…and by **counting on to find the difference** (small difference):

211 – 198 = 13



'The difference between 198 and 211 is 13.'

Introduce the **expanded written method** with the calculation presented both horizontally and vertically (in columns).
Use two-digit numbers when introducing this method, initially:

78 – 23 = 55

$$\begin{array}{r} 70 + 8 \\ -\underline{20 + 3} \\ 50 + 5 = 55 \end{array}$$

'Partition numbers into tens and ones/units. Subtract the ones, and then subtract the tens. Recombine to give the answer.'

NB In this example exchange is not required.

You might replace the **+ sign** with the word **'and'** to avoid confusion.

This will lead into the **formal written method,** where no exchange is required**:**

$$\begin{array}{r} 7\ 8 \\ -\underline{2\ 3} \\ 5\ 5 \end{array}$$

Use the language of place value to ensure understanding:

'Eight subtract three is five, seventy subtract twenty is fifty. The answer is fifty five'

NB An empty number line would be an appropriate method for this calculation but use two- digit numbers to illustrate the formal written method initially.

Use the **expanded written method** where **exchange** is required:

$73 - 27 = 46$

$70 + 3$ becomes $60 + 13$

$-\ 20 + 7$ $-\ 20 + 7$

$40 + 6 = 46$

73 is partitioned into 60+13 in order to calculate 73-27

NB children will need to practise partitioning numbers in this way.
Base- ten apparatus (e.g. Dienes) could be used to support this.

When children are confident with the expanded method introduce the **formal written method,** involving decomposition/exchange**:**

$73 - 27 = 46$

$$\begin{array}{r} {}^{6}\,{}^{13} \\ 7\ \cancel{3} \\ -\ 2\ 7 \\ \hline 4\ 6 \end{array}$$

Use the language of place value to ensure understanding.
'We can't subtract seven from three, so we need to exchange a ten for ten ones to give us 60 + 13.'

Use base ten apparatus to support understanding.

If children are confident, extend the use of the **formal written method** with numbers over 100, returning to the expanded method first, if necessary.

$235 - 127 = 108$

$$\begin{array}{r} {}^{2}\,{}^{15} \\ 2\ \cancel{3}\ \cancel{5} \\ -\ 1\ 2\ 7 \\ \hline 1\ 0\ 8 \end{array}$$

Use the language of place value to ensure understanding.
In this example it has only been necessary to exchange from the tens column.

Use base ten apparatus to support understanding.

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Subtraction - Year Four

- **Subtract numbers with up to 4 digits using the formal written method of columnar subtraction where appropriate**

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to teach the use of **empty number lines** with three and four digit numbers, as appropriate.

Continue to develop the formal written method of subtraction by revisiting the **expanded method** first, if necessary.

258 - 73 = 185

$$\begin{array}{r} 200 + 50 + 8 \\ -\quad 70 + 3 \\ \hline \end{array} \quad \text{becomes} \quad \begin{array}{r} 100 + 150 + 8 \\ -\quad 70 + 3 \\ \hline 100 + 80 + 5 \end{array} = 185$$

You might replace the **+ sign** with the word **'and'** to avoid confusion.

Children will need to practise partitioning in a variety of ways.
Use base ten apparatus (e.g. Dienes) to support understanding.

This leads to the **formal written method**, involving exchange…

$$\begin{array}{r} {\scriptstyle 1\ \ 15} \\ 2\ \cancel{5}\ 8 \\ -\quad 7\ 3 \\ \hline 1\ 8\ 5 \end{array}$$

Use the language of place value to ensure understanding.
In this example it has been necessary to exchange from the hundreds column.

Further develop by subtracting a three-digit number from a three-digit number:

637 – 252 = 385

Ensure that children are confident in partitioning numbers in this way.

$$\begin{array}{r} 600 + 30 + 7 \\ -\ 200 + 50 + 2 \\ \hline \end{array} \qquad \begin{array}{r} 500 + 130 + 7 \\ -\ 200 + 50 + 2 \\ \hline 300 + 80 + 5 \end{array} = 385$$

This leads to a **formal written method**:

```
   5 13
   6 3 7
 - 2 5 2
   3 8 5
```

Use the language of place value to ensure understanding and use base-ten apparatus, if necessary.

When children are confident, develop further with four- digit numbers and decimal numbers (in the context of money and measures).

3,625 – 1,219 = 2,406

```
     1 15
   3 6 2 5
 - 1 2 1 9
   2 4 0 6
```

£56·75 - £34·80 = £21·95

```
     5  17
   5 6· 7 5
 - 3 4·8 0
   2 1·9 5
```

Ensure that the decimal points line up**.**

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Year Five - Subtraction

- **Subtract whole numbers with more than 4 digits, including using formal written method (columnar subtraction)**

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to teach the use of **empty number lines** with larger numbers and decimal numbers, as appropriate.

Continue to develop the **formal written method for subtraction** with three-digit and four-digit numbers (see Y4 guidance), returning to an expanded method and using base ten apparatus, if necessary.

503 – 278 = 225

$$\begin{array}{r} 500+0+3 \\ -\,200+70+8 \\ \hline \end{array} \qquad \begin{array}{r} 400+90+13 \\ -\,200+70+8 \\ \hline 200+20+5 \end{array}$$

In this example 503 has to be partitioned into 400+90+13 in order to carry out the subtraction calculation.
Ensure that children are confident in partitioning numbers in this way.

This leads into the **formal written method** (there is potential for error in this example):

$$\begin{array}{r} {}^{4}\,{}^{9}\,{}^{13} \\ \cancel{5}\,\cancel{0}\,\cancel{3} \\ -\;2\;7\;8 \\ \hline 2\;2\;5 \end{array}$$

There are no tens in the first number (503) so we have to exchange a hundred for 10 tens before we can exchange a ten for ten ones/units

Consider the use of base ten apparatus to support understanding.

NB It would be appropriate to discuss the use of **mental calculation methods** with an example like this one, i.e. would an empty number line be a more efficient method for these numbers?

When children are confident, extend with larger numbers and decimal numbers.

12,731 – 1,367 = 11,364

```
      6 12 11
   1 2 7 3 1
 - 1 3 6 7
   1 1 3 6 4
```

In this example it has been necessary to exchange from the tens and the hundreds columns.

£166·25 – £83·72 = £82.53

```
    16  5 12
 1 6 6· 2 5
- 8 3· 7 2
  8 2· 5 3
```

Ensure the decimal points line up.

Continue to practise and apply the formal written method with larger numbers (numbers with more than four-digits) including decimal numbers.

NB If, at any time, children are making significant errors, return to the previous stage in calculation and provide calculations where only one exchange is required.

Year Six - Subtraction

No objectives have been included in the programmes of study explicitly related to written methods for subtraction in Y6.

However, there is an expectation that children will continue to practice and use **the formal written method for larger numbers and decimals** and use these methods when solving problems, when appropriate (see previous years' guidance for methods).

Our aim is that by the end of Y6 children **use mental methods (with jottings)** when appropriate, but for calculations that they cannot do mentally, they use an efficient **formal written method** accurately and with confidence.

Stages in Multiplication

Multiplication – Early Stages (EYFS)

Children will engage in a wide variety of songs and rhymes, games and activities. In practical activities and through discussion they will begin to solve problems involving doubling.

'Three apples for you and three apples for me. How many apples altogether?'

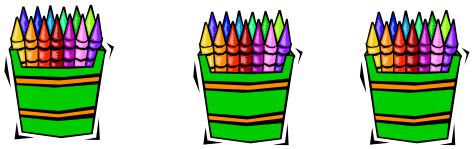
Multiplication – Year One

- **Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays**
- **Count in multiples of twos, fives and tens** (to the 10th multiple)

Children will count repeated groups of the same size in practical contexts. They will use the vocabulary associated with multiplication in practical contexts.

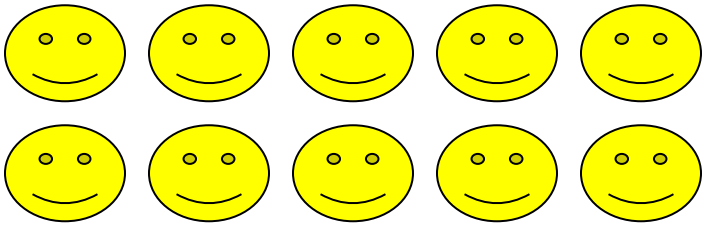
Use this to solve **practical problems** that involve combining groups of 2, 5 or 10 e.g. socks, fingers and cubes.

'Five pairs of socks.
How many socks altogether? 2, 4, 6, 8, 10'

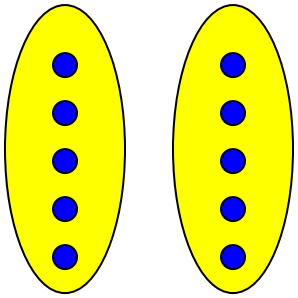


'Three pots of ten crayons. How many crayons altogether? 10, 20, 30'

Use **arrays** to support early multiplication



'Five groups of two faces. How many faces altogether? 2, 4, 6, 8, 10'
'Two groups of five faces. How many faces altogether? 5, 10'



'2 groups of 5'

'How many altogether?'

'5 + 5 = 10'

Double five is ten

Continue to solve problems **in practical contexts** and develop the language of early multiplication (but **not** the multiplication sign until Y2), with appropriate resources, throughout Y1.

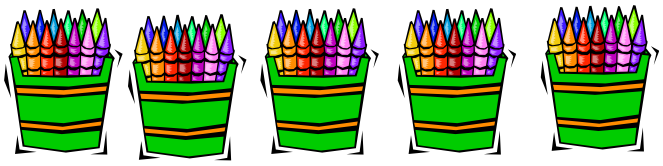
Multiplication - Year Two

- **Count in steps of 2, 3, 5 and 10 from 0**
- **Recall and use multiplication facts for the 2, 5 and 10 multiplication tables** (up to the 12th multiple)
- **Calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (×) and equals (=) signs**
- **solve problems involving multiplication, using materials, arrays, repeated addition, mental methods, and multiplication facts, including problems in contexts**
- **show that multiplication of two numbers can be done in any order (commutative)**

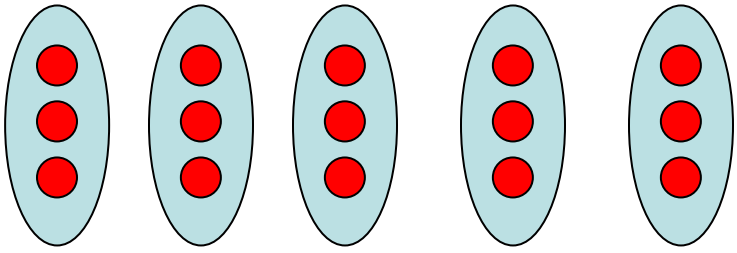
NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Children will use a range of vocabulary to describe multiplication and use practical resources, pictures, diagrams and the **x sign** to record.

Combining Groups (repeated addition):



'5 pots of 10 crayons'
'How many crayons altogether?'
'10, 20, 30, 40, 50'
'10 + 10 + 10 + 10 + 10 = 50'
'5 groups of 10' '5 times ten'
'5 x 10 = 50'



'5 groups of 3' '3, 6, 9, 12, 15'
'3 + 3 + 3 + 3 + 3 = 15' '5 x 3 = 15'

Use **arrays** to support multiplication:

6 x 5 = 30

‘How many stars altogether?’

‘5 + 5 + 5 + 5 + 5 + 5 = 30 ‘

‘6 rows of 5’

‘6 groups of 5’

‘5 groups of 6’

‘6 x 5 = 30’

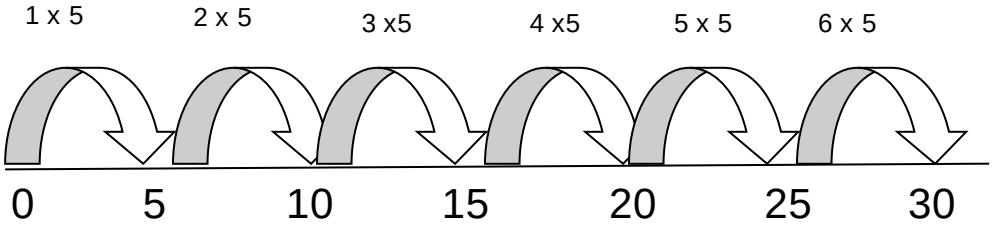
‘5 x 6 = 30’

Show that multiplication of two numbers can be done in any order

Use an **empty number line** to count on**:**

6 x 5 = 30

‘5, 10, 15, 20, 25, 30’



Make the link to repeated addition:

‘5 + 5 + 5 + 5 + 5 + 5 = 30
Six jumps of five equals 30
6 x 5 = 30’

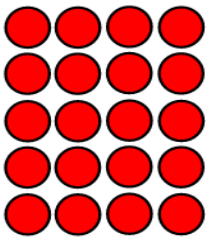
NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Multiplication - Year Three

- **Recall and use multiplication facts for the 3, 4 and 8 multiplication tables** (continue to practise the 2, 5 and 10 multiplication tables)
- **Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental methods and progressing to a formal written method**

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to use **number lines** and **arrays** to support multiplication, as appropriate.



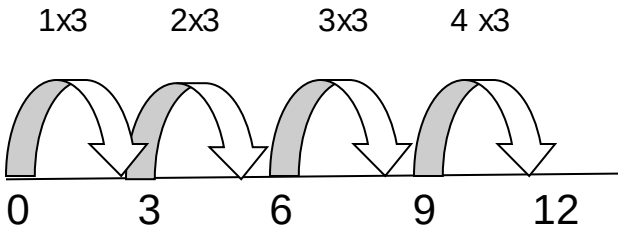
4 x 5 = 20

5 x 4 = 20

Use an **empty number** line to count on:

4 x 3 = 12

'0, 3, 6, 9, 12'



Partitioning method for multiplication of a teen number by a one-digit number:

13 x 5 = 65 (Partition 13 into 10 + 3)

10 x 5 = 50
3 x 5 = 15

50 + 15 = 65

Grid Method for multiplication of a teen number teen number by a one- digit number:

13 x 8 = 104

X	10	3
8	80	24

80 + 24 = 104

'Partition 13 into 10 + 3 then multiply each number by 8. Add the partial products (80 and 24) together.'

This will lead into **expanded short multiplication:**

13 x 8 = 104

10 + 3
x 8
2 4 (3 x 8)
+ 8 0 (10 x 8)
1 0 4

Include an addition symbol when adding partial products.

Refine the recording in preparation for formal short multiplication**:**

13 x 8 = 104

```
  1 3
x   8
-----
  2 4   (3 x 8)
+ 8 0   (10 x 8)
-----
1 0 4
-----
```

Use the language of place value to ensure understanding.

Include an addition symbol when adding partial products.

Formal short multiplication:

```
  1 3
x   8
-----
1 0 4
-----
  2
```

Ensure that the digit 'carried over' is written under the line in the correct column.

Use the language of place value to ensure understanding.

Continue to develop the formal written method of multiplication throughout Y3 using teen- numbers multiplied by a one-digit number.

If children are confident progress to multiplying other two-digit numbers by a one-digit number e.g. 24 x 3; 35 x 5 (see Y4 guidance).

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Multiplication- Year Four

- **Recall multiplication facts for multiplication tables up to 12 × 12**
- **Multiply two-digit and three-digit numbers by a one-digit number using formal written layout**

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Further develop the **grid method** for two-digit numbers multiplied by a one- digit number.

36 x 4 = 144

X	30	6
4	120	24

120 + 24 = 144 (add the partial products)

Expanded short multiplication (two-digit number by a one-digit number):

36 x 4 = 144

30 + 6
X 4
24 (4 x 6 = 24)
+ 120 (4 x 30 = 120)
144

Include an addition symbol when adding partial products.

Refine the recording in preparation for formal short multiplication:

36
x 4
24 (4 x 6)
+120 (4 x 30)
144

Short multiplication (formal method) of a two–digit number multiplied by a one-digit number:

36 x 4 = 144

```
  3 6
x   4
-----
1 4 4
  2
```

Use the language of place value to ensure understanding.
Ensure that the digit 'carried over' is written under the line in the correct column.

Continue to practise the formal method of short multiplication of a two-digit number by a one -digit number throughout Y4.

When children are confident, continue to develop short multiplication with three- digit numbers multiplied by a one-digit number.

If necessary, return to the grid method and/or expanded method first:

127 x 6 = 762

x	100	20	7
6	600	120	42

600 + 120 + 42 = 762 (add the partial products)

This leads to the expanded short multiplication method:

127 x 6 = 762

```
  1 2 7
x     6
-------
    4 2   (6 x 7)
+ 1 2 0   (6 x 20)
  6 0 0   (6 x 100)
-------
  7 6 2
```

Short multiplication (formal method) of a three-digit number multiplied by a one-digit number:

```
1 2 7
x   6
-----
7 6 2
1   4
```

Use the language of place value to ensure understanding.

Ensure that the digits 'carried over' are written under the line in the correct column.

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Multiplication - Year Five

- **Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers**

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Build on the work covered in Y4 with the **formal method of short multiplication** (two-digit or three-digit numbers multiplied by a one-digit number).

329 x 7 = 2,289

```
   3 2 7
 x     7
 -------
 2 2 8 9
   1 4
```

Ensure that the digits 'carried over' are written under the line in the correct column.

1,256 x 4 = 5,024

```
1 2 5 6
x     4
---------
5 0 2 4
1 2 2
```

When children are confident introduce **multiplication by a two-digit number**.
If necessary, return to the grid method and/or expanded method first.

Grid method (two-digit number multiplied by a two-digit number):

23 x 13 = (20 + 3) x (10 + 3) = 299

X	**20**	**3**
10	200	30
3	60	9

```
  2 3 0
+   6 9
-------
  2 9 9
```

Add the partial products (200 + 30) + (60 + 9) = 299

Expanded long multiplication (two-digit number multiplied by a two-digit number):

23 x 13 = 299

```
  2 3
x 1 3
    9   (3 x 3)
  6 0   (3 x 20)
+ 3 0   (10 x 3)
2 0 0   (10 x 20)
2 9 9
```

This leads into…

Compact long multiplication (formal method):

23 x 13 = 299

```
  2 3
x 1 3
+ 6 9   (3 x 23)
2 3 0   (10 x 23)
2 9 9
```

Use the language of place value to ensure understanding.

Add the partial products.

56 x 27 = 1,512

```
    5 6
  x 2 7
    3 9 4 2     (7 x 56)
+1 1 1 2 0      (20 x 56)
1 5 1 2
  1
```

Use the language of place value to ensure understanding.

In this example there are digits that have been 'carried' over in the partial products.

The prompts (in brackets) can be omitted if children no longer need them.

When children are confident with long multiplication extend with a three-digit number multiplied by a two-digit number:

124 x 26 = 3,224

$$\begin{array}{r} 1\,2\,4 \\ \times\quad 2\,6 \\ \hline 7^{1}4^{2}4 \quad (6 \times 124) \\ +\,2\,4\,8\,0 \quad (20 \times 124) \\ \hline 3\,2\,2\,4 \\ \,1\;1 \end{array}$$

Use the language of place value to ensure understanding.

Add the partial products.

The prompts (in brackets) can be omitted if children no longer need them.

Extend with short and long multiplication of decimal numbers (initially in the context of money and measures), returning to an expanded method first, if necessary (see Y6 guidance).

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Multiplication - Year Six

- **Multiply multi-digit numbers (including decimals) up to 4 digits by two-digit whole numbers**

NB Ensure that children are confident with the methods outlined in the previous years' guidance before moving on.

Continue to practise and develop the **formal short multiplication** method and the **formal long multiplication** method with larger numbers and decimal numbers throughout Y6. Return to an expanded forms of calculation initially, if necessary. **(See Y5 guidance).**

The formal written method of short multiplication:

3,256 x 4 = 13,024

```
  3256
x    4
-----
13024
  122
```

The formal written method of long multiplication:

53·2 x 24 = 1,276·8

```
   53·2
x   24
------
  2112·8    (53·2 x 4)
 1064·0     (53·2 x 20)
------
 1276·8
```

The prompts (in brackets) can be omitted if children no longer need them.

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Our aim is that by the end of Y6 children use **mental methods (with jottings)** when appropriate, but for calculations that they cannot do mentally, they use an efficient **formal written method** accurately and with confidence.

Stages in Division

Division – Early stages (EYFS)

Children will engage in a wide variety of songs and rhymes, games and activities. In practical activities and through discussion they will begin to solve problems involving halving and sharing.

Share the apples between two people.

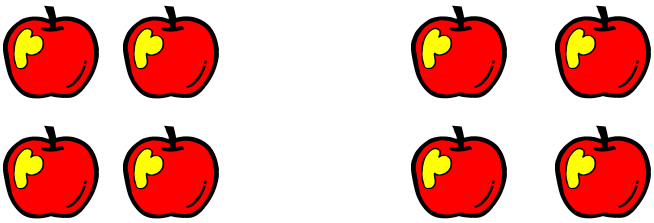
'Half of the apples for you and half of the apples for me.'

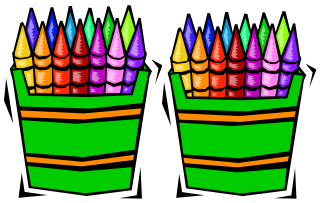
Division - Year One

- **Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays**
- **Count in multiples of twos, fives and tens** (to the 10th multiple)

Children will start with practical **sharing** using a variety of resources.
They will share objects into **equal groups** in a variety of situations.
They will begin to use the vocabulary associated with division in practical contexts.

'Share these eight apples equally between two children. How many apples will each child have?'

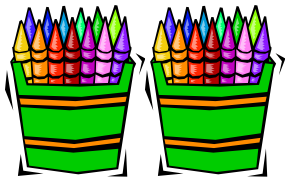




'Share 20 crayons between 2 pots.'

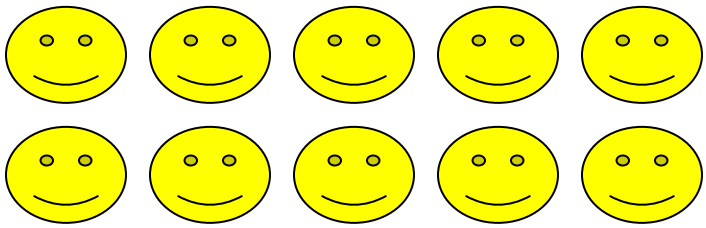
'How many crayons are in each pot?'

Children will move from **sharing** to **grouping** in a practical way

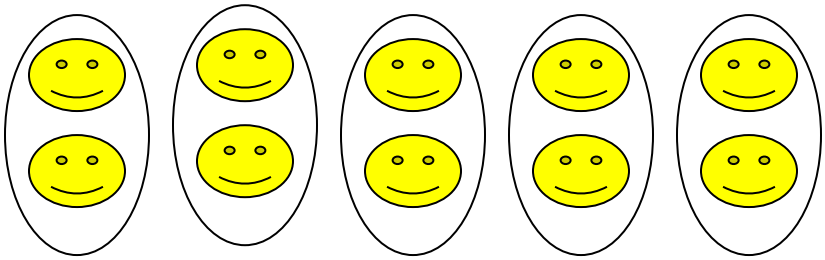


'Put 20 crayons into groups of 10. How many pots do we need?'

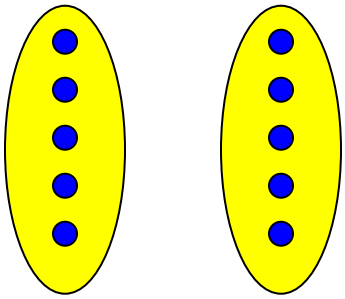
Use **arrays** to support early division



'How many faces altogether? How many groups of two?'



'Five groups of two'



'How many groups of 5?'
'10 shared equally between 2 people'
'Half of ten is five'

Continue to solve problems in **practical contexts** throughout Y1, and develop the language of early division (but **not** the division sign until Y2), with appropriate resources.

Division - Year Two

- **Count in steps of 2, 3, 5 and 10 from 0 (forward and backward)**
- **Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables**
- **Calculate mathematical statements for division within the multiplication tables they know and write them using the division (÷) and equals (=) signs**
- **Solve problems involving division, using materials, arrays, repeated subtraction, mental methods, and multiplication and division facts, including problems in contexts**

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

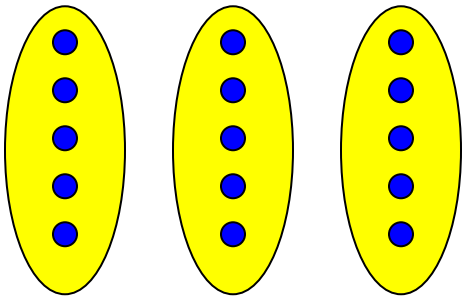
Children will use a range of vocabulary to describe division and use practical resources, pictures, diagrams and the **÷ sign** to record, using multiples that they know.

Sharing and grouping:

'30 crayons shared equally between three pots.' (Sharing)
'We have 30 crayons and put ten crayons in each pot. How many pots do we need?' (Grouping)

'30 divided by 10 = 3'
'30 divided by 3= 10'

30 ÷ 10 = 3
30 ÷ 3 = 10



'How many groups of 5?'
'15 shared equally between 3 people is…?'

'15 divided by 3 equals 5'
'15 divided by 5 equals 3'

15 ÷ 5 = 3
15 ÷ 3 = 5

Using arrays to support division

15 ÷ 5 = 3
15 ÷ 3= 5

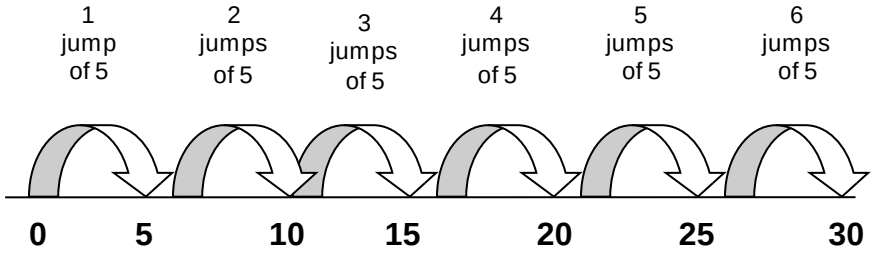
How many groups of 3?
How many groups of 5?
15 shared between 3 people is…?
15 shared between 5 people is…?

15 divided by 5 = 3
15 divided by 3 = 5

When children are ready, use an **empty number** line to count forwards to make the link with multiplication:

30 ÷ 5 = 6

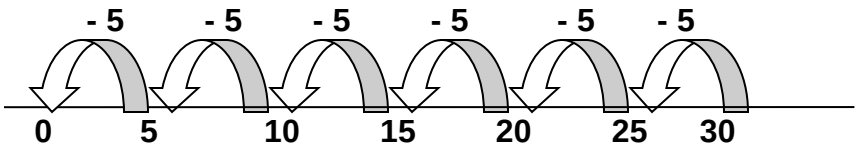
'How many jumps of five make thirty?'



Also jump back to make the link with repeated subtraction:

30 ÷ 5 = 6

'How many groups of five?'



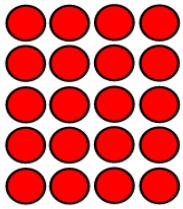
NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Year Three – Division

- **Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables** (continue to practise the 2, 5 and 10 multiplication tables)
- **Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers, using mental and progressing to a formal written method**

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to use practical resources, pictures, diagrams, number lines, arrays and the ÷ sign to record, using multiples that they know, as appropriate (see Y2 guidance).



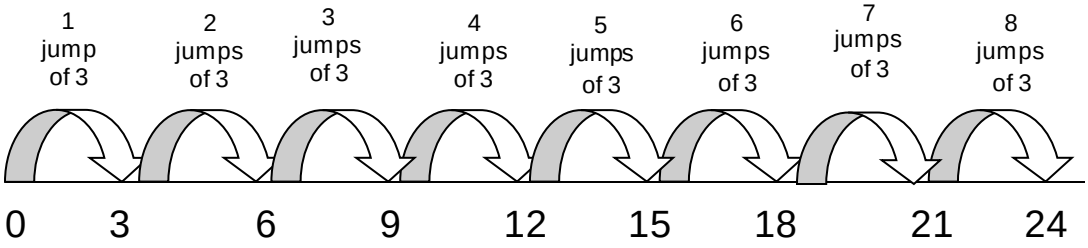
20 ÷ 5 = 4

20 ÷ 4 = 5

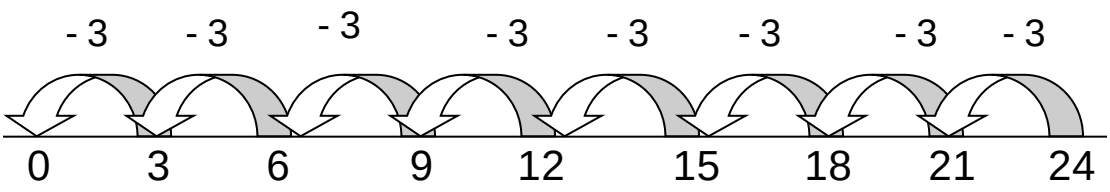
Use an empty number line to count forwards…

24 ÷ 3 = 8

'How many threes are there in 24?'



…also jump back from 24 to make the link with repeated subtraction.



Introduce the **formal layout** using multiplication/division facts that the children know:

$24 \div 3 = 8$

This can also be recorded as…

$$3 \overline{)\,24}\,\,8$$

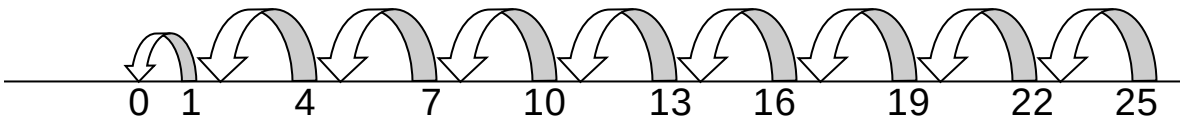
'Twenty four divided by three equals eight.'
'How many threes are there in twenty four?'

Begin to determine **remainders**, using known facts e.g. recognise that $13 \div 4$ will have a remainder of 1; $13 \div 4 = 3$ r 1

NB Remainders are not specifically referred to until Y5 in the National Curriculum. However, as an understanding of remainders are required for children working at 'greater depth' in Y2 (see KS1 Interim Assessment Framework), this may be an appropriate point to introduce them.

$25 \div 3 = 8$ r1

Eight jumps of three and one left over.'



Alternatively you could jump forwards in multiples of three from zero to twenty four ('and one more makes 25')

$$3 \overline{)\,25}\,\,8 \text{ r } 1$$

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Year Four- Division

- **Recall multiplication and division facts for multiplication tables up to 12 × 12**
- **Use place value, known and derived facts to divide mentally**
- **Divide two-digit and three-digit numbers by a one-digit number using formal written layout (not explicitly stated in the programmes of study but implied in the non-statutory guidance)**

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to write and calculate mathematical statements for division using the multiplication tables that the children know e.g.

$32 \div 8 = 4$

Continue using the **formal written layout** for division using known multiplication tables:

$$8\overline{)32}\quad 4$$

'How many eights are there in thirty two?'

Continue using the formal written layout, including examples with **remainders**:

$33 \div 8 = 4\ r1$

$$8\overline{)33}\quad 4\ r\ 1$$

This could be modelled using an empty number line to ensure understanding, if needed (See Y3 guidance)

Division using partitioning (two- digit numbers divided by a one –digit number):

65 ÷ 5 = 13

65 = 50 + 15 Partition 65 into 50 and 15

50 ÷ 5 = 10
15 ÷ 5 = 3
10 + 3 = 13

NB Children will need to practise partitioning in a variety of ways.

98 ÷ 7 = 14

98 = 70 + 28 Partition 98 into 70 and 28

70 ÷ 7 = 10
28 ÷ 7 = 4
10 + 4 = 14

This could be modelled on an empty number line to further develop understand.

Record the partitioning method using the formal layout

98 ÷ 7 = 14

‘We have partitioned 98 into 70 and 28 (90 = 70 + 28).

Seven ‘goes into’ 70 ten times and seven ‘goes into’ 28 four times.
Ten add four equals 14’

$$7\overline{)\,70 + 28}\quad 10 + 4 = 14$$

This will lead into the **formal written method of short division:**

98 ÷ 7 = 14

Use the vocabulary of place value to ensure understanding and make the link to partitioning.

$$7\overline{)\,9\,^{2}8}\quad 1\ 4$$

Continue to practise the formal method of short division throughout Y4.

If children are confident develop further, by dividing three-digit numbers by a one-digit number using the formal method of short division (first without remainders and then with remainders)**.**

$132 \div 6 = 22$

$$\begin{array}{r} 22 \\ 6\overline{)\,13^{1}2} \end{array}$$

Use the language of place value to ensure understanding.

Make the link to the partitioning method

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Year Five - Division

- **Divide numbers with up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context**

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to practise the formal written method of short division with whole number answers…

184 ÷ 8 = 23

$$\begin{array}{r} 2\ 3 \\ \hline 8\,\big)\,1\ 8^{2}4 \end{array}$$

Use the language of place value to ensure understanding.

Make the link to the partitioning method (see Y4 guidance).

…and with remainders:

432 ÷ 5 = 86 r2

$$\begin{array}{r} 8\ 6\ \mathrm{r}\ 2 \\ \hline 5\,\big)\,4\ 3^{3}2 \end{array}$$

The remainder can also be expressed as a fraction, 2/5 (the remainder divided by the divisor): 432 ÷ 5 = 86 2/5

Continue to practise, develop and extend the formal method of short division, with and without remainders.
Interpret and express remainders according to the context when solving problems.

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Year Six – Division

- **Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context**
- **Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context**

NB Ensure that children are confident with the methods outlined in the previous years' guidance before moving on.

Continue to practise the **formal method of short division**, with and without remainders, using the language of place value to ensure understanding (see Y5 guidance).

652 ÷ 8 = 81 r4

$$8\overline{)\,6\,5\,{}^{1}2}\qquad 8\ 1\ r\ 4$$

The remainder can be expressed as a fraction, 4/8 (the remainder divided by the divisor), which can be simplified to 1/2.

The **formal method of short division** can sometimes be used to divide by a two-digit number:

496 ÷ 11 = 45 r1

$$11\overline{)\,4\,9\,{}^{5}6}\qquad 4\ 5\ r\ 1$$

The remainder can also be expressed as a fraction, $\frac{1}{11}$ (the remainder divided by the divisor).

The same calculation can be solved using a **formal method of long division** (by repeated subtraction using multiples of the divisor):

```
      4 5 r 1
    ___________
11 ) 4 9 6
   - 4 4 0   (40 x 11)
     5 6
   -   5 5   (5 x 11)
       1     (remainder)
```

Multiples of the divisor (11) have been subtracted from the dividend (496)

'40 (lots of 11) + 5 (lots of 11) = 45 (lots of 11)'

'1 is the remainder'

Answer: $45\frac{1}{11}$

Standard short division does not help with the following calculation.
However, it can be solved using **long division**:

144 ÷ 16 = 9

```
      9
    _____
16 ) 1 4 4
   - 6 4   (4 x 16)
     8 0
   - 6 4   (4 x 16)
     1 6
   - 1 6   (1 x 16)
       0
```

Multiples of the divisor (16) have been subtracted from the dividend (144)

'4 (lots of 16) + 4 (lots of 16) + 1 (lot of 16) = 9 (lots of 16).

The answer is nine.

There is no remainder'

Children will need to select the most effective method for each calculation/problem they meet, including whether to use a formal written method of short division or a formal written method of long division.

432 ÷ 15 = 28 r12

```
      2 8  r12
    ___________
15 ) 4 3 2
   3 0 0   (20 x 15)
   1 3 2
   1 2 0   (8 x 15)
     1 2   (remainder)
```

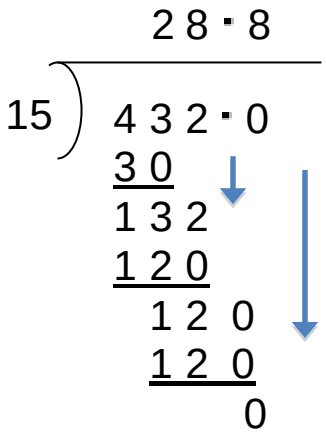
Multiples of the divisor (15) have been subtracted from the dividend (432)

'20 (lots of 15) + 8 (lots of 15) = 28
12 is the remainder'

The remainder can also be expressed as a fraction, 12/15 which can be simplified to 4/5, or as a decimal, 0.8 (See next example)

This is an alternative way of recording formal long division:

$432 \div 15 = 28{\cdot}8$



Only teach this method when children are completely secure with the previous method.

The remainder is expressed as a decimal.

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Our aim is that by the end of Y6 children use **mental methods (with jottings)** when appropriate, but for calculations that they cannot do mentally, they use an efficient **formal written method** accurately and with confidence.

NOTES